

SYNTHESIS OF 500-MB. HEIGHT AND TEMPERATURE DATA BY CONSIDERATION OF SURFACE PRESSURE AND TEMPERATURE AND THE BEHAVIOR OF DENSITY WITH HEIGHT¹

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ABSTRACT

A procedure for synthesizing 500-mb. height and temperature data over oceanic areas using only sea level data as an input is developed.

The procedure is tested by comparing the synthetic data to observed 500-mb. data at 4YN and 4YP and by comparing 500-mb. synthetic data charts to 500-mb. charts based on observed data. The North Pacific is chosen for the latter test.

The 500-mb. height variations given by synthetic data compare favorably with the observed height variations at 4YN and 4YP. While certain errors are apparent in the synthetic 500-mb. chart it does show many of the features of the chart produced from observed data.

1. INTRODUCTION

In 1943 Doporto [1] outlined a procedure for constructing 8-km. charts for the pressure field using only surface data and the fact that density at the 8-km. level remains almost constant with time and space. Doporto claimed an accuracy of the order of the radiosonde accuracy of that period.

To my knowledge this technique never acquired operational status, in spite of its simplicity and claimed accuracy. This failure to achieve operational status is quite surprising, since a need certainly existed then, as well as now, for acquisition or synthesis of upper-air data to fill the large gaps in the upper-air observing network such as exist over oceanic areas. Speculation as to the causes of this failure suggests that several factors may have contributed: (1) accuracy was not as claimed, (2) the 8-km. level was perhaps a little higher than the operational ceiling of most of the aircraft of that period, (3) publication was in a somewhat obscure governmental document, (4) meteorologists schooled in isobaric analysis found constant height charts cumbersome, (5) security and communication problems of wartime.

Whatever the cause of the failure of Doporto's technique to gain operational status, it is hoped that this article describing the derivation and application of a modified form of Doporto's work will serve to bring about a reconsideration of the merits of this method of upper data synthesis.

2. DEVELOPMENT OF THEORY

Consider a hydrostatic atmosphere with a linear lapse rate of temperature. Density can be expressed as a function of geopotential altitude by

$$\rho(z) = \rho_0 \left[\frac{T_0 + \alpha z}{T_0} \right]^{-\{1+g/(R\alpha)\}} \quad (1)$$

where $\rho(z)$ is density at altitude z ; ρ_0 is density at the base; T_0 is the temperature at the base; α is the vertical gradient of temperature, i.e., the negative of the lapse rate; g is the standard value of gravity (9.80665 m. sec.⁻²); R is the gas constant for dry air. For the standard atmosphere, density is expressed as a function of geopotential altitude by a similar expression:

$$\rho'(z) = \rho'_0 \left[\frac{T'_0 + \alpha' z}{T'_0} \right]^{-\{1+g/(R\alpha')\}} \quad (1a)$$

where the primes refer to values of the parameters in the standard atmosphere.

It is known, because of observation [2], [3], [4] and theoretical [5], [6] considerations, that it is most likely that the curves representing the distribution of density with height intersect at some height above the surface, and that this height is probably near 8 km. If we denote this altitude of intersection as z^* , then

$$\rho'(z^*) = \rho(z^*)$$

or

$$\rho'_0 \left[1 + \frac{\alpha' z^*}{T'_0} \right]^{-\{1+g/(R\alpha')\}} = \rho_0 \left[1 + \frac{\alpha z^*}{T_0} \right]^{-\{1+g/(R\alpha)\}} \quad (2)$$

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Taking the natural logarithm of the expression, expanding the terms of the form $\ln(1+x)$ to $(x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \dots)$, retaining only the first two terms of the expansion, and regrouping them we reduce (2) to

$$\ln \frac{\rho'_0}{\rho_0} = \left\{ \frac{\alpha' z^*}{T'_0} - \frac{\alpha'^2 z^{*2}}{2T'^2_0} + \frac{gz^*}{RT'_0} - \frac{g\alpha' z^{*2}}{2RT'^2_0} \right\} - \frac{\alpha z^*}{T_0} + \frac{\alpha^2 z^{*2}}{2T^2_0} - \frac{gz^*}{RT_0} + \frac{g\alpha z^{*2}}{2RT^2_0} \quad (3)$$

Note that, except for z^* , the term in the above expression enclosed by braces depends only on the values in the standard atmosphere. For simplicity of expression this term will be referred to as K .

Equation (3) may now be rearranged and solved by the quadratic formula. Of the two solutions only the solution with the negative sign before the radical is appropriate in the lower regions of the atmosphere.

$$\alpha = \frac{1}{2} \left[\frac{2T_0}{z^*} - \frac{g}{R} - \sqrt{\left(\frac{2T_0}{z^*} - \frac{g}{R} \right)^2 - \frac{8T^2_0}{z^{*2}} \left(K + \ln \frac{\rho'_0}{\rho_0} - \frac{gz^*}{RT_0} \right)} \right] \quad (4)$$

This solution then represents the vertical gradient of temperature which must exist in the arbitrarily selected atmosphere with linear lapse rate in order for the density in this atmosphere to be identical to that in the standard atmosphere at height z^* above the surface. From (4) we see that $\alpha = \alpha(T_0, z^*, g, R, \rho_0)$ but since $\rho_0 = \rho_0(p_0, T_0, R)$ and R and g are constants (assuming dry air and using geopotential altitude)

$$\alpha = \alpha(T_0, p_0, z^*)$$

where p_0 is pressure at the base.

The value chosen for z^* should be near to that found in nature. In this note z^* is defined as the value of the mean scale height of the two atmospheres in question, i.e., $(H_p + H_r)/2$ where

$$H_p = \frac{RT_0}{g}$$

hence

$$\alpha = \alpha(p_0, T_0)$$

This is not necessarily the best value to choose for z^* in terms of accuracy of the 500-mb. data, but it was used throughout this study.

Now consider the distribution of pressure p with height in the atmosphere defined by p_0 , T_0 , and α :

$$p(z) = p_0 \left[\frac{T_0 + \alpha z}{T_0} \right]^{-g/(R\alpha)}$$

Taking the natural logarithm of the above expression and expanding as for density give

$$-\ln \frac{p}{p_0} = \frac{gz}{RT_0} - \frac{g\alpha z^2}{2RT^2_0}$$

Solving for z by the quadratic formula (retaining only the root with the positive sign preceding the radical) we may determine the height of a given pressure surface (p) above the base

$$z(p) = T_0 \left[\frac{1}{\alpha} + \sqrt{\frac{1}{\alpha^2} + \frac{2R}{g\alpha} \ln \frac{p}{p_0}} \right]$$

Temperature being a linear function of height, by definition in this atmosphere, it may be expressed simply as

$$T(z) = T_0 + \alpha z$$

Thus, the height and temperature of the 500-mb. surface as well as the value of α for this atmosphere may be defined by only T_0 and p_0 and may be determined by the above expressions. These computations have been made for values of p_0 ranging between 970 and 1040 mb., and for values of T_0 ranging between -10° and 35° C. The results have been used to construct nomograms so that the synthetic values of 500-mb. height and temperature may be easily determined for given values of p_0 and T_0 within the above range. Sketches of these are given in figures 1 and 2.

The logical test for accuracy of the synthetic data is the comparison to data observed by the radiosonde network, but before this test is applied perhaps the assumption made in deriving the above expressions should be examined to determine possible sources of error.

First, the isopycnic level was assumed known; this is not necessarily the case. While it is true that the height of the isopycnic level is normally found in the vicinity of one scale height above the surface, Riehl [7] found several cases (all associated with deep low pressure systems) in which no isopycnic level existed. Such occurrences had been suggested by theory (Whitehead and Blick [5]).

Another simplifying assumption made was that temperature is a linear function of height. A perfectly linear temperature profile, of course, almost never exists in the atmosphere over such depth, and great departures can occur, as with subsidence and frontal inversions.

These two sources of error are believed to be the primary causes of the departure of the synthetic data values from observed values, although the possibility exists that the radiosonde data, with a standard deviation of height error of the order of 25 m. at 500 mb., may contribute to this departure.

3. COMPARISON OF SYNTHETIC TO OBSERVED 500-MB. DATA

Test data were subjected to two types of comparison to observed data. The computed 500-mb. heights were compared to the observed 500-mb. heights at ships 4YP (50° N., 145° W.) and 4YN (30° N., 140° W.), and the 500-mb. charts prepared from synthetic data alone were compared with charts prepared by the U.S. Weather Bureau for oceanic areas for which the analyst used all

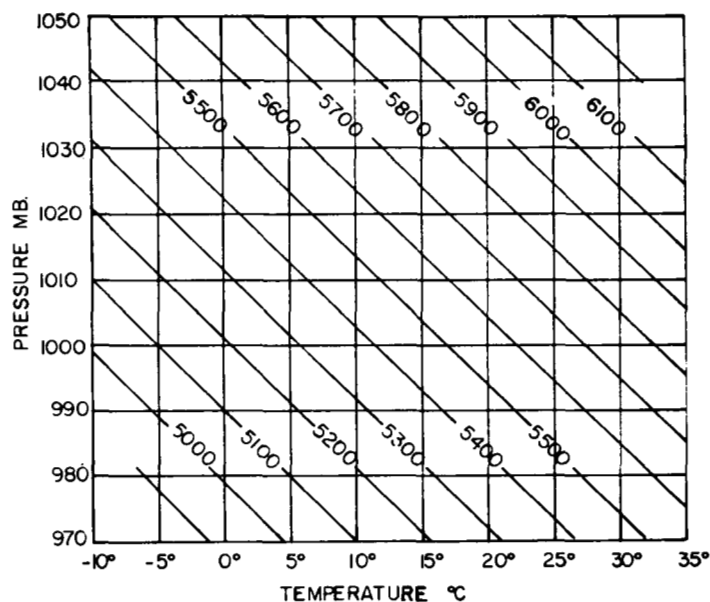


FIGURE 1.—Nomogram for determining synthetic 500-mb. height (in meters) as a function of surface pressure and temperature.

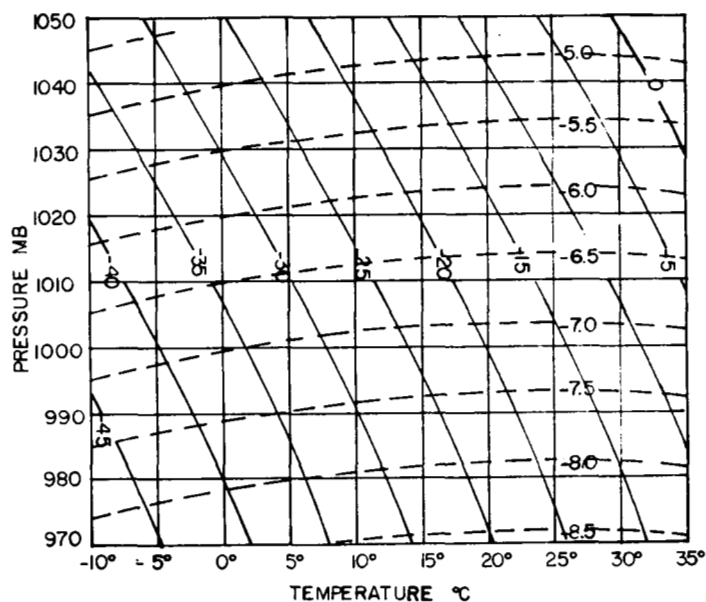


FIGURE 2.—Nomogram for determining synthetic 500-mb. temperature (solid sloping lines) and the synthetic linear vertical temperature gradient (dashed lines) as a function of surface pressure and temperature.

available observed data. The results will be discussed separately.

Comparison of synthetic and observed data at 4YP and 4YN.—Five days were chosen at random for each month for the year 1961. The 0000 GMT surface observations from the two ships were used to compute the synthetic 500-mb. heights and temperatures which were then compared to the observed values and the errors determined. (It was assumed there was no error in the radiosonde observations.) Figures 3 and 4 are histograms depicting the distribution of the error in 500-mb. height (z synthetic minus z observed) at 4YN and 4YP, respectively. The error distribution for the former is much more orderly than for the latter. For 4YN, synthetic data tend to give values too low for 500-mb. height. Indications are that this is due to the effect of the trade wind inversion at this station. Later it will be shown that synthetic data consistently give heights too low for the subtropics.

Because the wind field can be approximated from the gradient of the pressure field, the gradient of pressure is probably as important a parameter as the pressure height itself. A comparison of the height differences between 4YN and 4YP as indicated by radiosonde and by synthetic data is given by means of a scattergram (fig. 5). The distribution of points in this figure suggests that a regression curve could be drawn such that a useful tool for determining the height differences between 4YP and 4YN by means of synthetic data could be developed.

In order to compare the change in the 500-mb. height as indicated by radiosonde and by synthetic data, figures 6 and 7 were prepared. These figures show the indicated

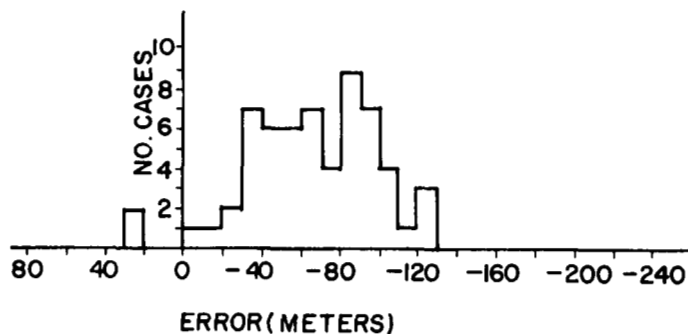


FIGURE 3.—Histogram of the synthetic 500-mb. height error at ship station 4YN, assuming radiosonde data are accurate.

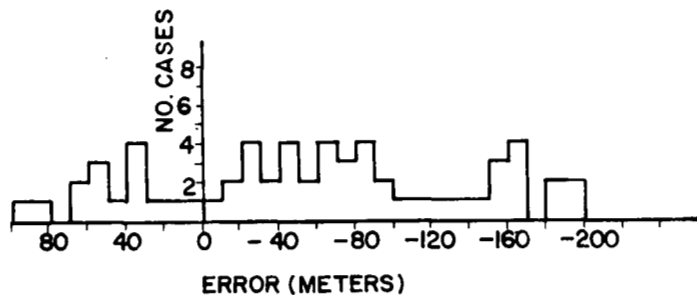


FIGURE 4.—Histogram of the synthetic 500-mb. height error at ship station 4YP, assuming radiosonde data are accurate.

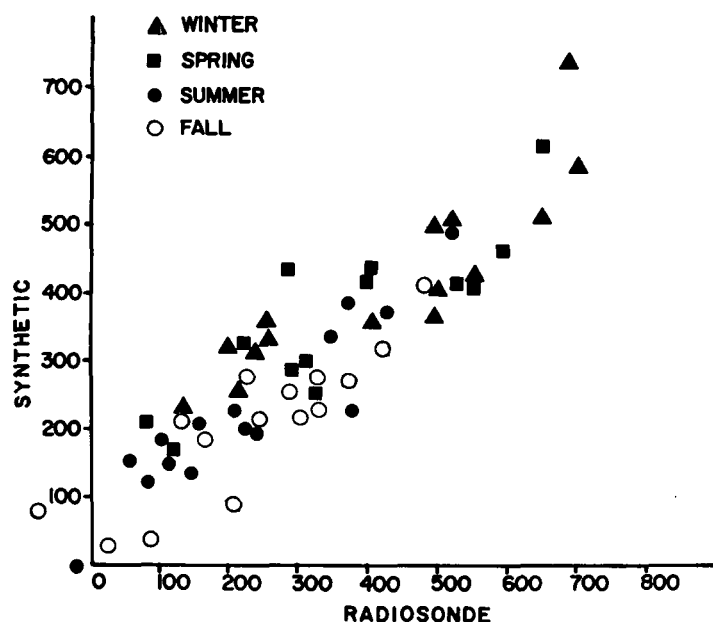


FIGURE 5.—Comparison of synthetic to observed 500-mb. height differences between 4YP and 4YN (in meters) for five days selected at random for each month of 1961.

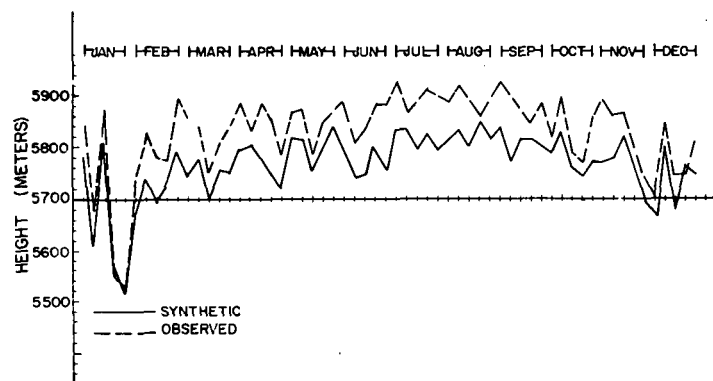


FIGURE 6.—Observed and synthetic 500-mb. heights for 4YN for five days selected at random for each month of 1961.

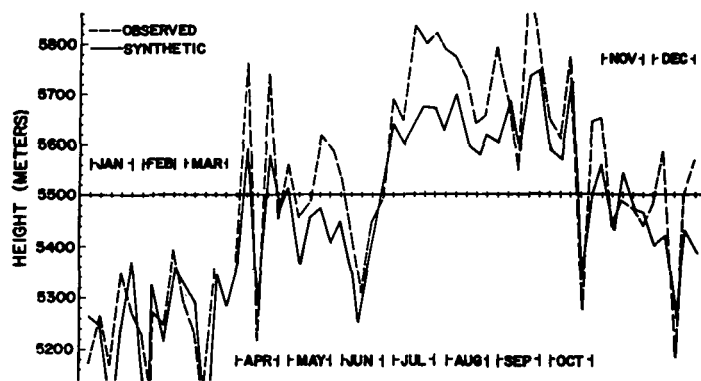


FIGURE 7.—Observed and synthetic 500-mb. heights for 4YP for five days selected at random for each month of 1961.

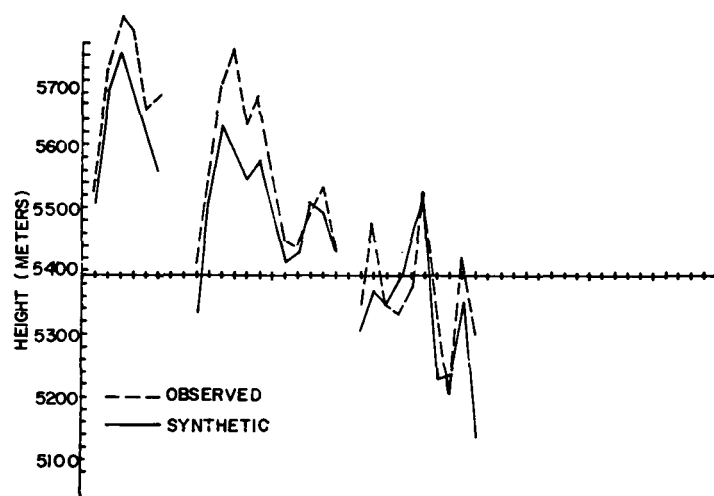


FIGURE 8.—Observed and synthetic 500-mb. heights for 4YP for consecutive days in March 1958.

height of the 500-mb. surface by the two methods as the seasons progress. Keep in mind that these are not consecutive days but randomly selected days. In general the amplitude of the perturbations indicated by the synthetic data is not as large as indicated by radiosonde, but a definite similarity between the patterns is apparent. An example of the comparison of the two methods for consecutive days is shown for 4YP in figure 8.

Comparison of 500-mb. charts drawn from synthetic data with those drawn from observed data.—February 17–20, 1959 was arbitrarily chosen as the test period for comparison of 500-mb. charts drawn from the two types of data (synthetic and observed). Data used to generate the synthetic data were surface temperature and pressure for ship and island stations plotted on the 1200 GMT Daily Series Synoptic Weather Maps [8] for those days; in a few cases coastal continental surface data were also used to generate the 500-mb. synthetic data. Upper-air data of any sort, climatology, and continuity were *not* used in producing the 500-mb. synthetic data charts.

For the sake of brevity, only one pair of the four pairs of resulting charts is shown here (figs. 9 and 10). Each pair of charts consisted of one 500-mb. chart depicting the height and temperature contours as determined by synthetic data and one 500-mb. chart representing a tracing of the 1200 GMT chart published in [8]. The latter are based on all available appropriate data (this of course becomes somewhat skimpy over oceans). For both charts the values of the pressure altitude contours (in meters) are given at the left extremity and the values of the isotherms (in degrees Celsius) at the right extremity.

Comparison of figures 9 and 10 brings out both the advantages and deficiencies of the synthetic data. The two figures have common features in: (a) the location of the short-wave troughs in the central and eastern Pacific, (b) the location of the Low near Shemya Island, (c) the

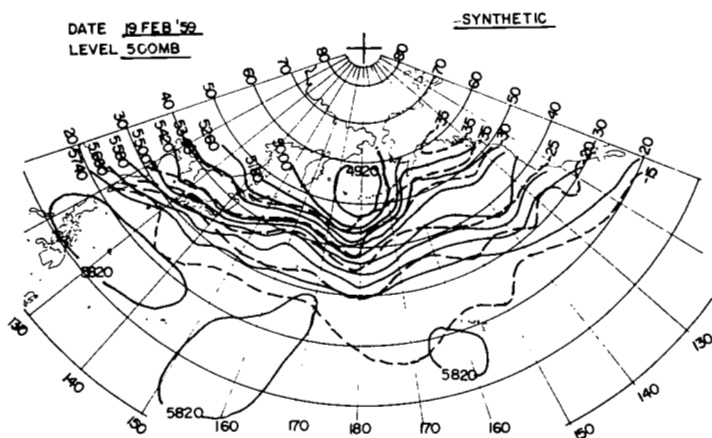


FIGURE 9.—Synthetic 500-mb. height and temperature pattern for February 19, 1959.

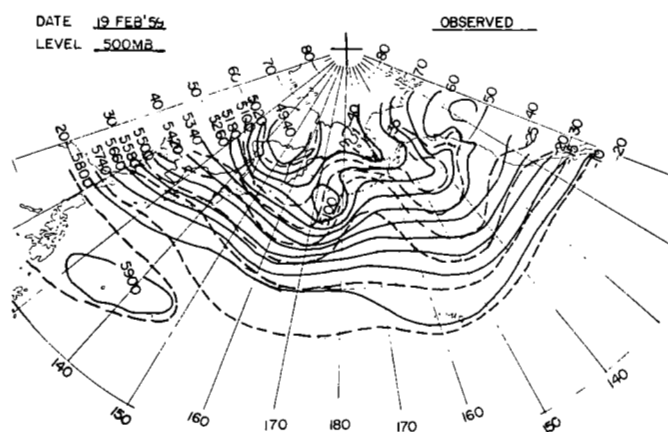


FIGURE 10.—Observed 500-mb. height and temperature pattern for February 19, 1959.

general area of strongest jet stream, and (d) the subtropical high pressure belt. Departures between the two patterns are also apparent: (a) The synthetic subtropical heights are too low (probably because of trade-wind inversion). (b) The synthetic short-wave troughs, while well defined, are somewhat to the east of their indicated positions on the observed data chart. (c) The synthetic jet stream appears a little to the north of the observed jet stream. (d) The synthetic isotherms have insufficient amplitude. (e) There are large departures from the observed pattern at the edge of the data. (This is caused to a large extent by ignoring the observed data over continents.)

Hindsight suggests that the Atlantic would have been a better test area for the synthetic data. There, the observed 500-mb. chart is probably more exact because of more radiosonde and more air traffic, and the discrepancies between the two series of charts might then be attributed almost entirely to error in synthetic data.

4. CONCLUSIONS

Synthetic 500-mb. data generated in the manner described in this note appear to be rather good approximations to the observed data. The fact that unmodified synthetic data (using no upper-air data) can be used to prepare a 500-mb. chart, which resembles as it does the chart prepared by observed data, suggests that this technique can have potential value.

It is not anticipated that synthetic data will be used in lieu of observations, but rather will complement them. There is little doubt that the daily synthetic charts prepared here could have been improved considerably in the vicinity of coasts by considering the observed upper data over the continents. The use of observed oceanic upper data also would have improved the synthetic data charts. Conversely, it appears that the consideration of a modified form of synthetic data in the preparation of the charts

based on observed data would have improved the observed data charts, and that is essentially the purpose of synthetic data.

Above, a "modified form" of synthetic data was mentioned. What this modification is, and how it is performed, are questions yet unanswered. The error, i.e., the difference between synthetic and observed data, seemed to be orderly in some respects. As examples: the subtropical synthetic heights were in general too low, the amplitude of the synthetic height oscillations was less than that of those observed, and synthetic troughs appeared to the east of observed troughs. This orderly nature of the error suggests that correction factors, which consider latitude, season, location of surface observation with respect to the surface systems, and perhaps other factors, may be applied to the raw synthetic data to reduce the error to an acceptable value.

The features common between figures 9 and 10 were, in general, also common between each of the other pairs of charts. The other pairs also showed departures in the subtropical heights and the short-wave trough position similar to those between figures 9 and 10. The jet stream positions on the other pairs of charts were in better agreement than those shown here, however.

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CORRECTION

Vol. 93, August 1965, p. 505: In table 1, the entries for altitude for Oklahoma City, Okla. and Greensboro N. S. should be interchanged.